Condensed Matter Theory — problem set 12

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Problem 32: Anderson-Morel model

So far we have solved the gap equation for an attractive phonon-induced interaction, but in a metal there is also the repulsive Coulomb part (cf. lecture),

$$V_{\text{eff}}(\boldsymbol{q} = \boldsymbol{k} - \boldsymbol{k}') = \frac{4\pi e^2}{q^2 + q_{\text{TF}}^2} + \frac{4\pi e^2}{q^2 + q_{\text{TF}}^2} \frac{\omega_q^2}{\omega^2 - \omega_q^2}. \tag{1}$$

(a) Show that the gap equation for a gap $\Delta(\xi)$ which depends only on energy can be written just below T_c in the form (why is there a minus sign?)

$$\Delta(\xi) = -\nu_0 \int d\xi' \, V(\xi, \xi') \frac{\tanh(\beta\xi'/2)}{2\xi'} \Delta(\xi'). \tag{2}$$

(b) One can model the effective interaction with a repulsive Coulomb and attractive electron-phonon interaction as

$$V(\boldsymbol{k}, \boldsymbol{k}') = V(\xi, \xi') = V_{\text{ee}}(\xi, \xi') + V_{\text{eph}}(\xi, \xi') \tag{3}$$

where

$$V_{\text{ee}}(\xi, \xi') = \begin{cases} V_0 > 0 & -W < \xi, \xi' < W, \\ 0 & \text{otherwise} \end{cases} \qquad V_{\text{eph}} = \begin{cases} -\delta < 0 & -\omega_D < \xi, \xi' < \omega_D, \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

W is the bandwidth of electrons and $\omega_D < W$ the Debye frequency. Choose an ansatz for the gap

$$\Delta(\xi) = \begin{cases} \Delta_1 & |\xi| < \omega_D, \\ \Delta_2 & \omega_D < |\xi| < W \end{cases} \tag{5}$$

and determine T_c from the gap equation. Interpret your result.

[*Hint*: Your result should have a form

$$T_c = 1.13\omega_D \exp[-1/\nu_0(\delta - V^*)], \qquad V^* = \frac{V_0}{1 + V_0 \ln(W/\omega_D)}.$$

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Problem 33: BCS spin susceptibility

(a) Show that the spin susceptibility in a BCS superconductor is given by

$$\frac{\chi}{\chi_P} = \frac{2}{3\pi^2}\frac{\varepsilon_F^0}{n}\int_0^\infty dp\, p^2 \left[-\frac{\partial f(E_p)}{\partial E_p}\right] \tag{6}$$

in terms of the Pauli spin susceptibility χ_P of a normal metal.
Hint: An infinitesimal external Zeeman field h couples to the electronic spin density operator $\sum_{\boldsymbol{k}}(c^\dagger_{\boldsymbol{k}\uparrow}c_{\boldsymbol{k}\uparrow} - c^\dagger_{\boldsymbol{k}\downarrow}c_{\boldsymbol{k}\downarrow})$; express the whole Hamiltonian in terms of the original Bogoliubov quasiparticles γ and compute the h derivative of $\langle M\rangle$ as $h \to 0$.

(b) Verify the following limits: $\chi = 0$ at $T = 0$ and $\chi = \chi_P$ at $T = T_c$. Interpret these results.

Problem 34: Critical temperature of a homogeneous 2D Bose gas

In the lecture we derived the critical temperature T_c below which a three-dimensional noninteracting Bose gas forms a Bose-Einstein condensate. Try to use the same reasoning to compute T_c for a *two-dimensional* homogeneous Bose gas: what is different? Compute μ for a given density and temperature (thermal wavelength). What does this imply for the value of T_c, and the existence of a BEC?