Quantum critical transport in the unitary Fermi gas

Tilman Enss (TU München)

ERG 2012, Aussois, 3 September 2012

Enss, PRA **86**, 013616 (2012)
Enss, Küppersbusch, Fritz, PRA **86**, 013617 (2012)
Enss and Haussmann, arXiv:1207.3103

Technische Universität München

Unitary Fermi gas

- Fermi gas with contact interaction

$$S = \int d^d x \, d\tau \sum_{\sigma=\uparrow,\downarrow} \psi_\sigma^* \left[\partial_\tau - \frac{\nabla^2}{2m} - \mu_\sigma\right] \psi_\sigma + g \, \psi_\uparrow^* \psi_\downarrow^* \psi_\downarrow \psi_\uparrow$$

- scattering amplitude (3d)

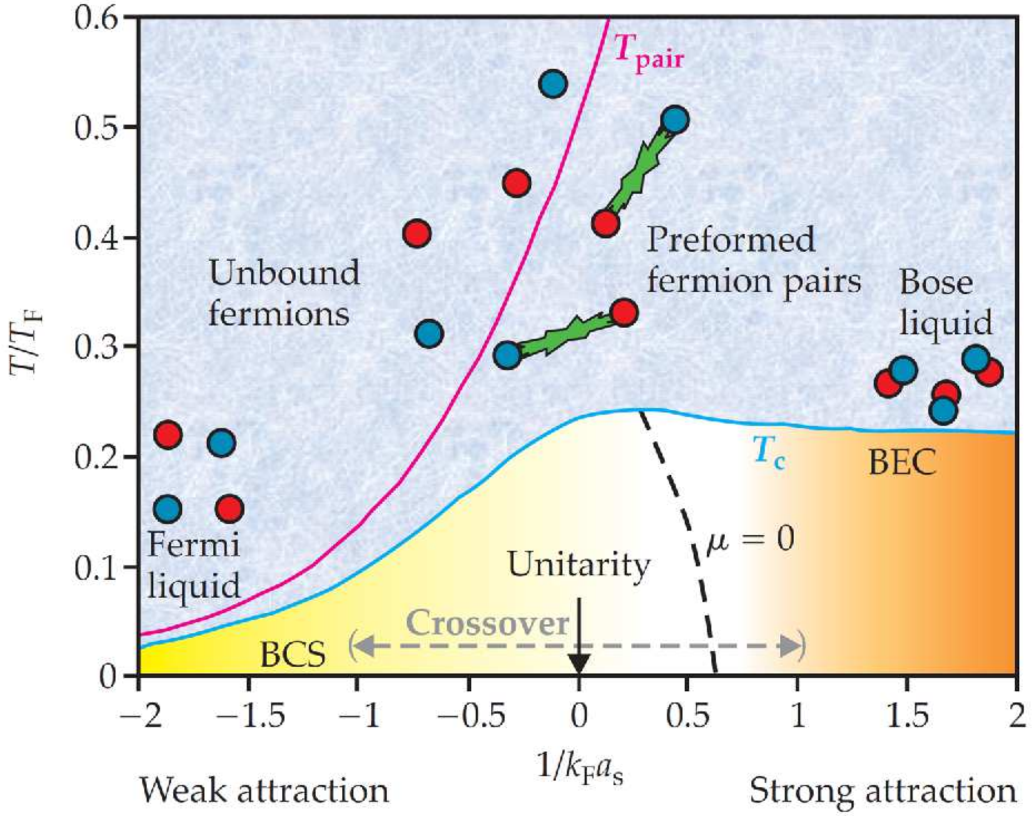
$$f(k) = \frac{1}{-1/a - ik + r_e k^2/2}$$

- strong scattering in unitary limit

$$1/a = 0: \quad f(k \to 0) = \frac{i}{k}$$

- universal for dilute system (broad resonance)

$$r_e \ll n^{-1/3}$$



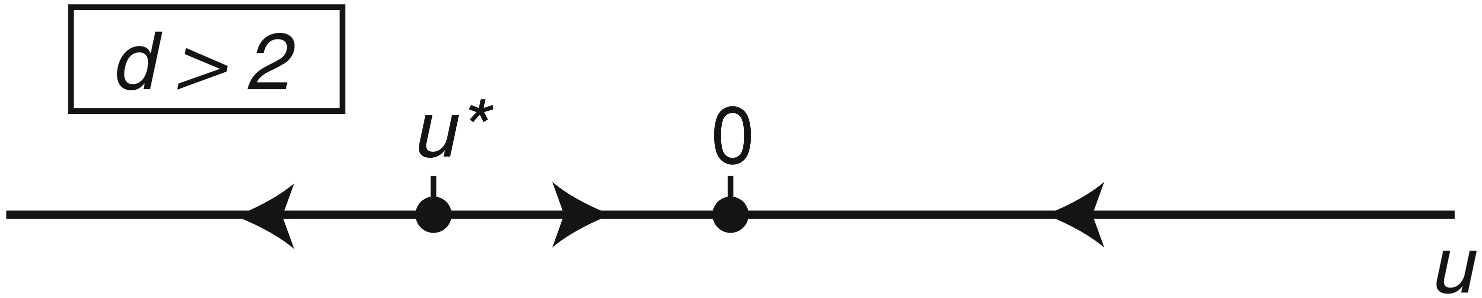
[Sa de Melo, Physics Today 2008]

Renormalization group

- vacuum (T=0, n=0): **exact** beta function [Nikolic, Sachdev; Diehl et al.]

$$\frac{dg}{d\ell} = (2-d)g - \frac{g^2}{2}$$

- 2<d<4: unstable fixed point g* (unitarity, Feshbach resonance)

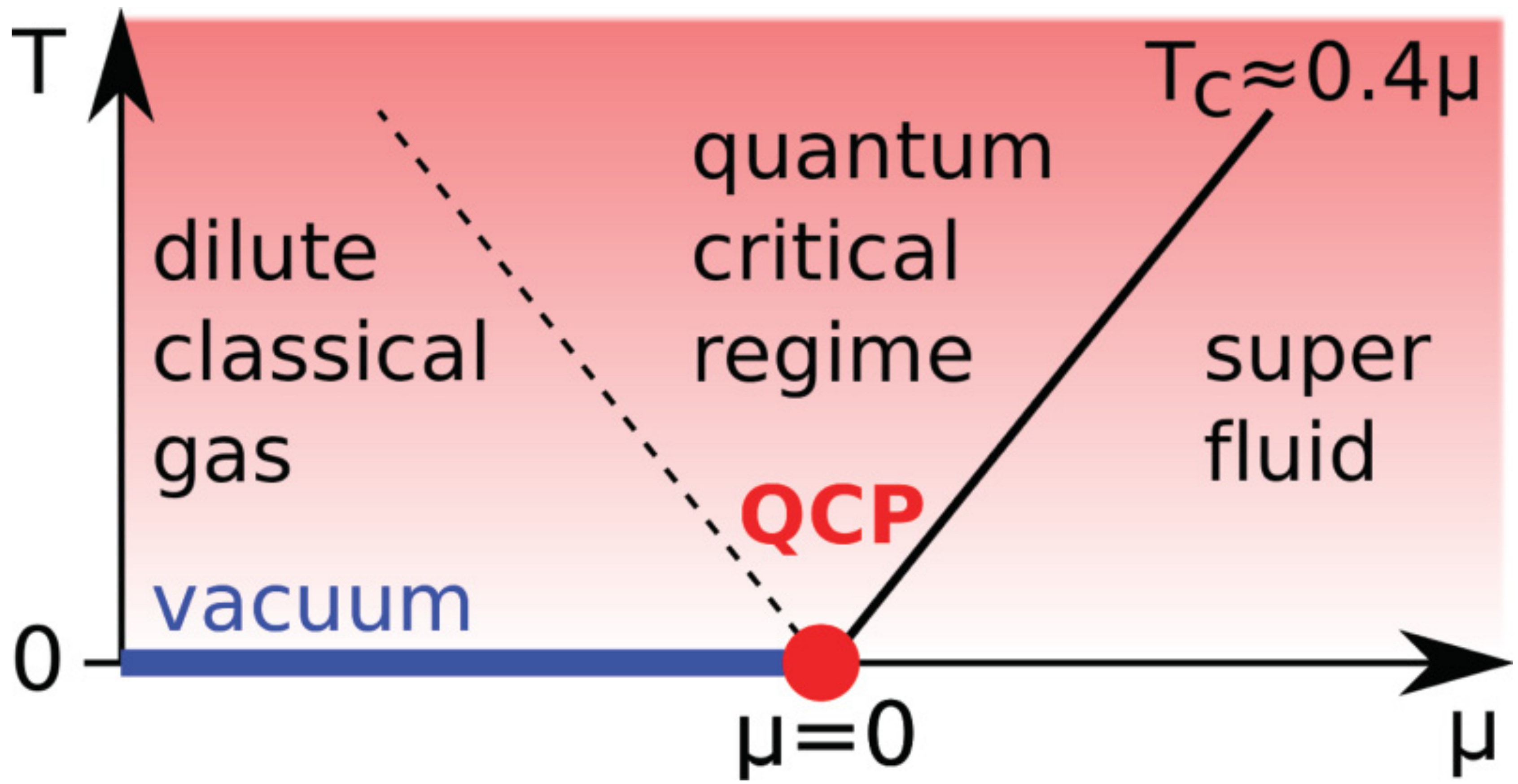


[Nikolic, Sachdev 2007]

- detuning 1/a is relevant perturbation

Quantum critical point

- resonant fixed point is Quantum critical point (QCP) [Nikolic, Sachdev]



[Enss 2012]

- density n is order parameter: vacuum for T=0, μ<0

Universal properties

- thermodynamic functions depend only on **μ/T** (“angle”)

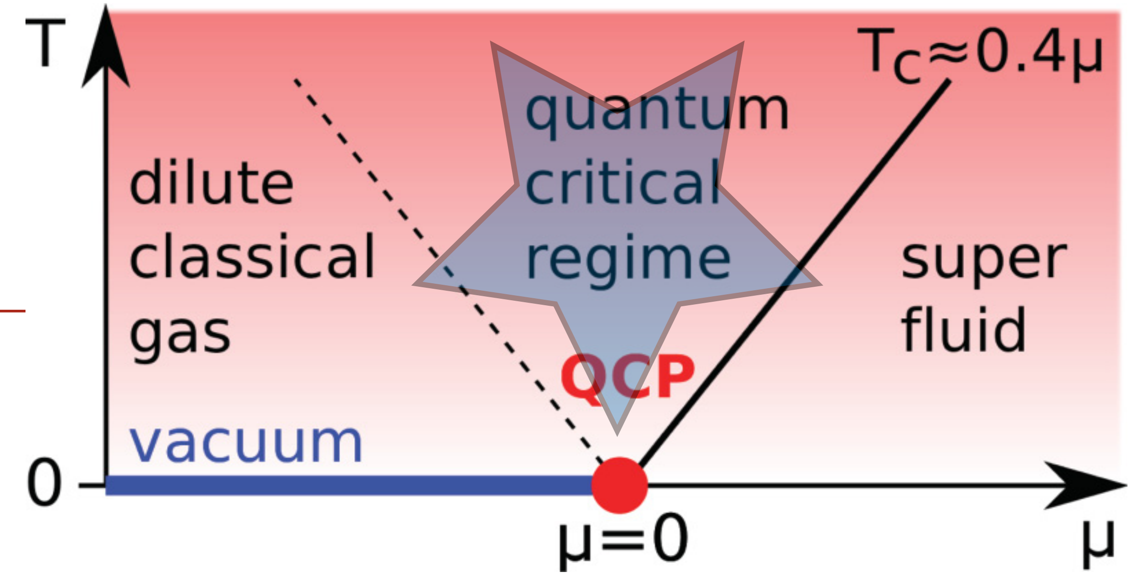
  e.g. equation of state $n = \lambda_T^{-3} f_n(\mu/T)$

- measured by Zwierlein group, Science 2012, computed using Bold Diagrammatic MC:

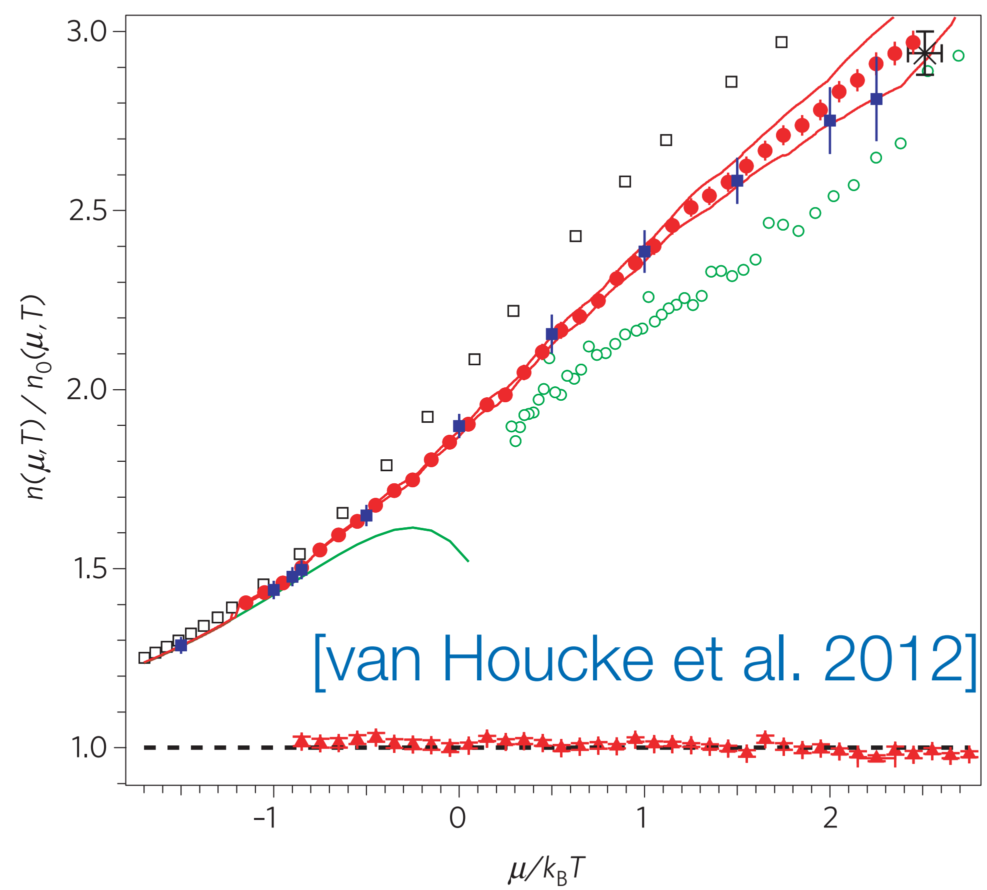
  agreement on percent level (benchmark)

- open: contact, imbalance, superfluid (in progr.) challenge: **transport**

  focus on **quantum critical regime:** $\lambda_T \approx n^{-1/3}$ quantum and thermal fluctuations equally important



[Enss 2012]



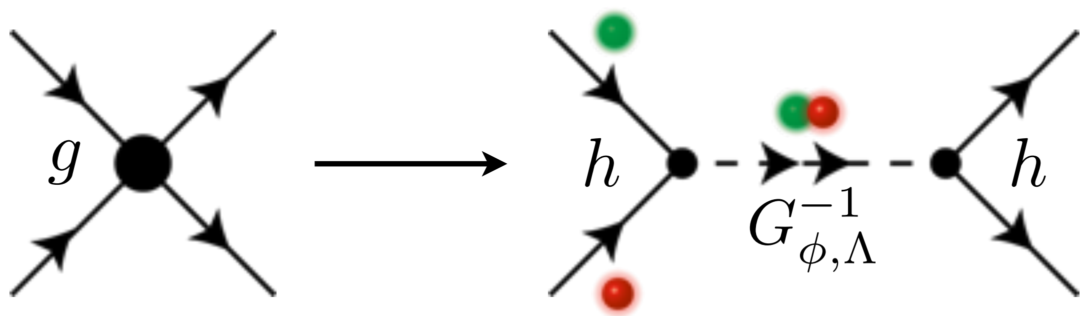
[van Houcke et al. 2012]

Effective action

- Hubbard-Stratonovich transformation in **Cooper channel:** exchange of virtual molecules (T-matrix)

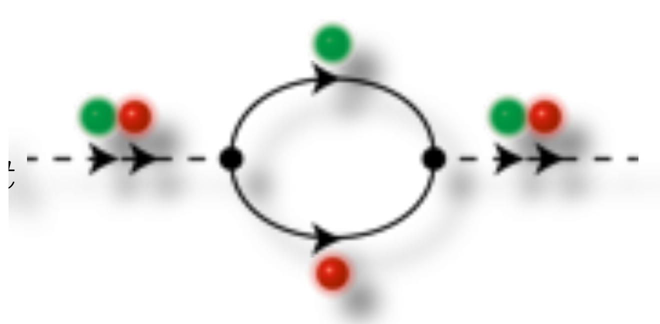
$$S = \int d^d x \, d\tau \sum_{\sigma=\uparrow,\downarrow} \psi_\sigma^* \left[\partial_\tau - \frac{\nabla^2}{2m} - \mu_\sigma\right]\psi_\sigma - \frac{1}{g_0}|\phi|^2 - h\psi_\uparrow^*\psi_\downarrow^*\phi - h\phi^*\psi_\downarrow\psi_\uparrow$$

(positive mass!)



- integrate out fermions: bosonic effective action

$$\mathrm{tr}\ln G_f[\phi] = b_2(q,\omega)\,|\phi|^2 + b_4(q_i,\omega_i)\,|\phi|^4 + \cdots$$



Vacuum (at QCP)

- 3d, T=μ=0: $b_2(q,\omega) \simeq \sqrt{\frac{q^2}{4m} - \omega - i0}$
- anomalous dimension:

$$\eta_\phi = 4 - d > 0, \quad \dim[\phi] = \frac{d + \eta_\phi}{2} = 2 \quad (2 < d < 4)$$

- n-point functions $b_n \sim q^{5-2n}$ all **scale marginally** [Enss 2012]
- no simple ϕ^4 theory (Hertz-Millis)
- in vacuum no feedback on 2-point function, dg/dl remains exact **but** possible limit cycles in 3-body sector can change ground state (Efimov physics, **Richard Schmidt's talk on Friday**) [Floerchinger, Schmidt, Moroz, Wetterich]

Finite density

- T>0, n>0: all higher n-point functions feed back into ϕ propagator, no obvious strategy to select diagrams (no small parameter)

- **strong coupling many-body problem:**
  - sample *all* diagrams (Bold Diagrammatic MC)
  - Luttinger-Ward (2PI): self-consistent propagators (1-loop skeleton diagrams)
  - functional RG: derivative expansion; full ω,q [Schmidt, Enss 2011]

- **large-N expansion:** [Nikolic, Sachdev]
  N flavors of ↑↓ fermions
  fermion loops: factor N
  pair propagators: factor 1/N (N=∞ free fermions)
  controlled expansion in orders 1/N
  extrapolate to physical case N=1
  details: `Enss, PRA 86, 013616 (2012)`

Results: thermodynamics

- leading order large-N expansion (NSR) at μ=0: [Enss 2012]

	Experiment	Large-N	LuttWard	BoldDiagMC
$n\lambda_T^3$	2.966(35) [8]	2.674	3.108 [26]	2.90(5) [9]
P $[nk_BT]$	0.891(19) [8]	0.928	0.863 [26]	0.90(2) [9]
s $[nk_B]$	2.227(38) [8]	2.320	2.177 [26]	2.25(5) [9]
C $[k_F^4]$		0.0789	0.084 [18]	0.080(5) [27]
η/s $[\hbar/k_B]$	1.0(2) [19, 28]	0.741	0.708 [18]	

- pressure and entropy within 3%, **contact density within 1%** of expt./BDMC:

$$C = m^2\langle\phi^*\phi\rangle = -\frac{m^2}{N}\int\frac{d^3k}{(2\pi)^3}\frac{d\omega}{\pi}b(\omega)\,\mathrm{Im}\,\mathcal{T}(k,\omega)$$

$$= 26.840\,128\,\frac{\lambda_T^{-4}}{N}\,.$$

- **good and efficient approximations available!**

Results: transport

- Boltzmann equation justified in large-N expansion;
  need in-medium T-matrix for consistency!

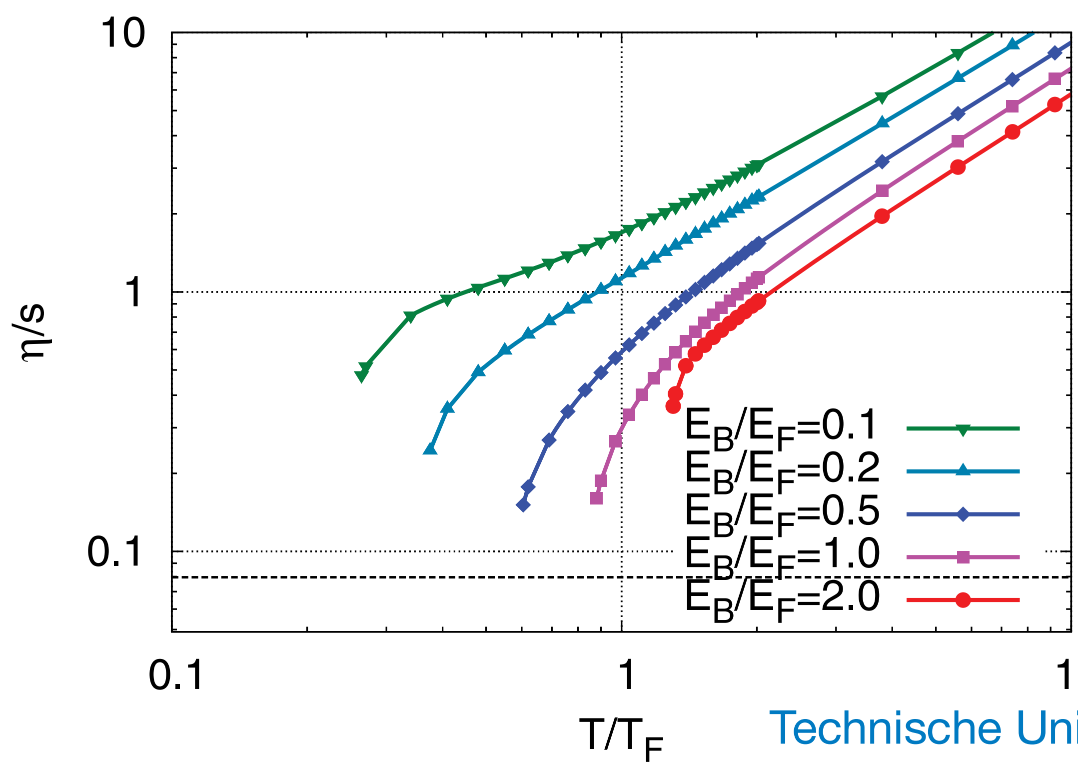
- transport in quantum critical regime: scaling with temperature

$$\eta \sim \hbar T^{3/2}, \quad s \sim k_B T^{3/2} \quad \longrightarrow \quad \frac{\eta}{s} = 0.74 \, \frac{\hbar}{k_B}$$

- viscosity η/s measures incoherent relaxation rate:

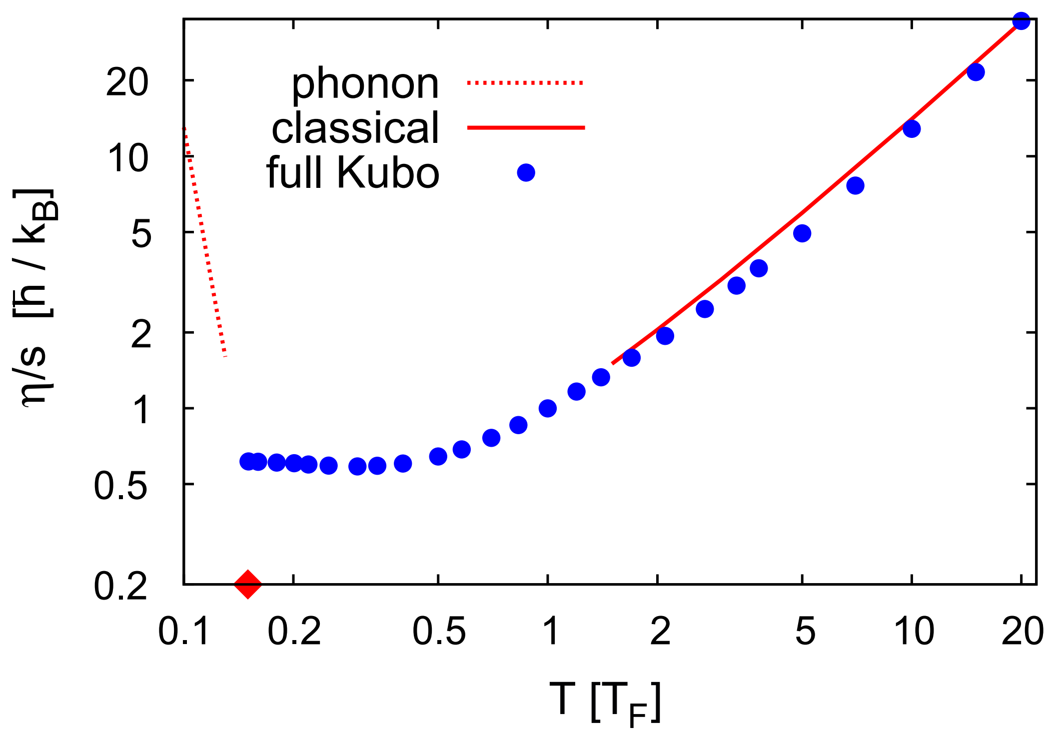
$$\frac{\hbar}{\tau_\eta} = 0.54 \, k_B T$$

- large-N in 2d: viscosity
  [Enss, Küppersbusch, Fritz,
  PRA **86**, 013617 (2012)]

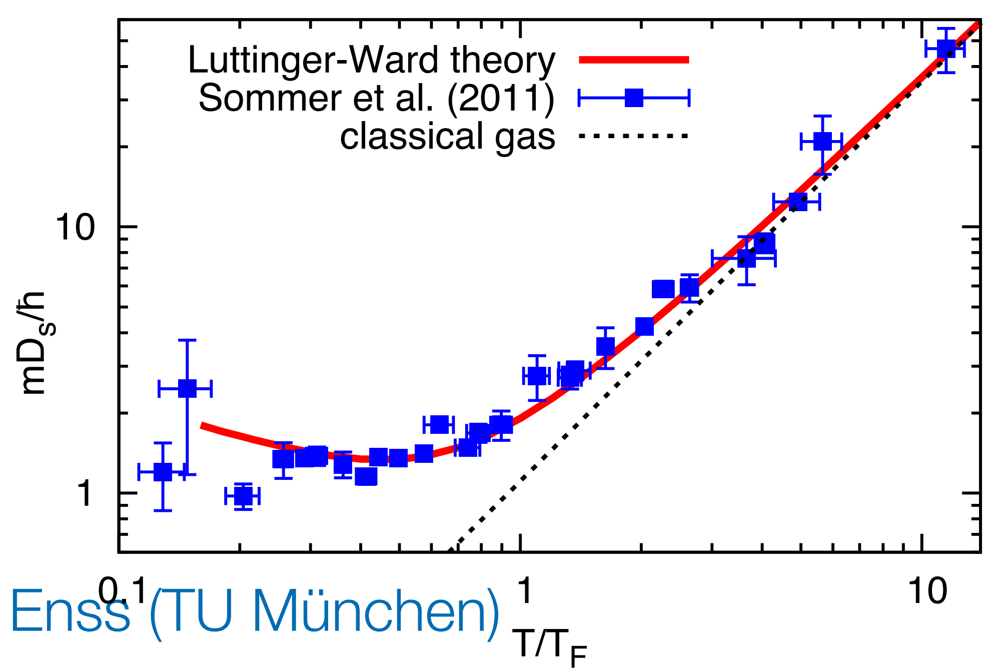


Results: transport

- Luttinger-Ward (2PI): self-consistent fermion and pair propagators



viscosity η/s:
Enss, Haussmann, Zwerger 2011



spin diffusion rate $\mathbf{D_s \sim \hbar/m}$
Enss, Haussmann, arXiv:1207.3103 (2012)

comparison with experimental data:
Sommer et al. (Zwierlein group), Nature 2011

Conclusions

- phase diagram of unitary Fermi gas governed by **quantum critical point**
- scaling analysis of effective action:
  infinity number of **marginal vertices**, approximations not obvious
- comparison with benchmark:
  large-N expansion, Luttinger-Ward (2PI), functional RG work well
- lesson for functional RG:
  integrate out fermions and bosons simultaneously; full ω,q dependence helps
- large-N accurately determines pressure, entropy, **contact;**
  transport calculations can explain recent experiments:
  quantum limited spin diffusion